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LABORATORY AND ATMOSPHERIC VORTEX INSTABILITIES

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ABSTRACT

Mesoscale atmospheric vortices such as dust devils, tornadoes and firewhirls may be modelled in the laboratory if the similarity requirement of equal advance angles of the upward swirling flow is preserved. Theoretical analysis shows that linear vortices are stable even for large pertubations near their base. However, they become unstable to small downstream perturbations which change the relative advance angle to about 68^{o}. Laboratory experiments which have preserved similarity to large ambient vortices have demonstrated destructive instability. They suggest that flow pertubations introduced by appropriate cloud seeding could destroy tornadoes.

INTRODUCTION

Perturbations in the flow field near a vortex line or ring can alter the vortex parameters considerably. Under certain conditions, the vortices can be made unstable by small lateral pertubations. Their characteristic size should be about the same as the diameter of the vertical vortex. An example of such an impulse is the change in the flow field as a result of the energy generated by seeding a tornado cloud along a horizontal line near the freezing level. Two kinds of atmospheric vortices can be studied both experimentally and analytically. They are: (1) the cylindrically symmetric line vortex and (2) the vortex ring.

McCormack et al. (1972) reported that a flaming vortex ring apparently became unstable as a result of combustion. The burning causes the vortex ring to expand, thereby altering the flow field so that ambient air is entrained into it. This behavior is unstable because an impulse dominates the motion (Scorer, 1958). Vortex rings are not a significant atmospheric phenomena.

However, the cylindrically symmetrical vortex is an important atmospheric phenomenon, occurring as dust-devil, tornado or firewhirl. It has been studied extensively, both theoretically and experimentally. One of the most successful vortex generators, constructed by Emmons and Ying (1967), used a rotating cylindrical screen to generate vorticity of liquid pool flames. They showed that the finite vortex core rotated as a solid body. Chigier et al. (1970) showed that the liquid fuel could be replaced by a

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gas source, which increased the control of the flames but had little effect on the vortex. Beer et al. (1971) also demonstrated the stability of this arrangement by using a variety of mechanical devices to introduce vorticity into the system. The rototating screen vortex generator has also been used to simulate atmospheric vortices (Bareilon, 1967).

This paper reports a study to establish similarity between atmospheric vortices in the laboratory and in the field, in terms of the advance angle of the flow around the exterior of the solid core vortex. This angle is important in determining the stability of the vortex.

The mathematical analysis that follows is for a vortex which does not have an internal source of energy, although this apparently is not an important factor. Most of the experiments were performed with fire whirls because theyare more easily photographed, although similar experimental results were obtained with a pool of warm water which also generated a visible line vortex. In addition, accurate, close-up photos of firewhirls were available (Palmer, 1970), a condition not met for dustwhirls and tornadoes.

THEORETICAL DISCUSSION

Breakdown of a line vortex is observed downstream from the point of origin of the vortex under a wide range of conditions. Kuchemann (1965) listed a wide variety of contexts in which this breakdown can occur. The process is one of sudden expansion of the axially symmetric stream tubes of the vortex over a distance on the order of the diameter of the vortex.

Theoretical studies of this phenomenon have been made by Benjamin (1962), Bess (1967), and others. The mathematical description of these vortices can be made in the context of a wide variety of geophysical phenomena, including fire whirls, dust-devils and tornadoes. The actual breakdown mechanism is a result of one of three processes: (1) hydrodynamic instability, (2) standing waves, or (3) a finite transition between two flow states such as a hydraulic jump.

These conditions can be approximated linearly, by using the non-viscous equations for axially symmetric rotating flow in circular cylindrical coordinates. The boundary conditions to be satisfied are that the horizonatal components of fluid velocity, v_θ and v_r, and the radial derivative of the vertical velocity, w, must be zero on the axis of the vortex.

The general dimensional Navier-Stokes of fluid motion for viscous, axially symmetric flow in circular cylindrical coordinates are:

$$\frac{u_\Theta}{r} + \frac{\partial u_\Theta}{\partial r} + \frac{\partial w}{\partial z} = 0$$

$$v_\Theta \frac{\partial v_\Theta}{\partial r} + w \frac{\partial v_\Theta}{\partial z} - \frac{v_r^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left\{\frac{\partial^2 v_\Theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\Theta}{\partial r} + \frac{\partial^2 v_\Theta}{\partial z^2} - \frac{v_\Theta}{r^2}\right\}$$

$$v_\Theta \frac{\partial v_r}{\partial r} + w \frac{\partial v_\Theta}{\partial z} + \frac{v_\Theta v_r}{r} = \nu\left\{\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2}\right\}$$

$$v_\Theta \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{-1}{\rho}\frac{\partial p}{\partial z} + \nu\left\{\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right\}$$

where r is the radius, Θ is the angle, z is the vertical, (v_r, v_Θ, w) are components of velocity along r, Θ, and z.

These equations can be simplified by assuming inviscid flow and a stream function Ψ where

$$v_\Theta = \frac{-1}{r}\frac{\partial \Psi}{\partial z}, \quad w = \frac{1}{r}\frac{\partial \Psi}{\partial r}, \quad \text{and} \quad rv_r = k(\Psi)$$

after Goldstein (1938)

A transformation of the form

$$y = \frac{r^2}{2} \quad , \quad h = v_\Theta r \quad , \quad \text{and} \quad k = v_r r$$

simplifies the equation of motion to

$$\frac{\partial^2\Psi}{\partial z^2} + 2y\frac{\partial^2\Psi}{\partial y^2} = -k\frac{dk}{d\Psi} + \frac{2y}{\rho}\frac{dp_o}{d\Psi}$$

where ρ is the density and p_o is the reference pressure. This equation can linearized by assuming a Taylor's series expansion in Ψ on the right hand side of the equation. Neglecting powers of Ψ higher than two leads to the equation:

$$\frac{\partial^2\Psi}{\partial z^2} + 2y\frac{\partial^2\Psi}{2y^2} + \Psi\left\{\beta_2 - 2\alpha_2 y\right\} = 2y\alpha_1 - \beta_1$$

where (α_i, β_i) are the coefficients in the series expansion.

Separation of variables by choosing

$$\Psi(y,z) = Y(y)Z(z)$$

leads to the two total differential equations

$$Y'' + \left(\frac{\mu^2}{2y}\right)Y = 0 \qquad \text{and} \qquad Z'' + (\lambda^2 - \mu^2)Z = 0$$

where μ is the separation constant.

SOLUTIONS

These equations have solutions which are products of exponential functions and Bessel functions of integer order one, and of the first and second kind. The boundary condition that the stream function on the outer boundary of the vortex (at radius r_c) be ($w_\infty r^2/2$) sets the requirement that the Bessel function of the first kind vanish at that point. This indicates that a change of the mode of flow is possible due to hydrodynamic instability. This condition is described by the requirement that the argument of the Bessel function ($2 v_c / w_\infty$) (where v_c is the speed at the edge of the vortex core) causes the Bessel function to be zero. This occurs when it is equal to 3.831...which corresponds to a breakdown swirl angle Ø of:

$$
\begin{aligned}
\emptyset &= \arctan(3.832/2) \\
&= 62.5^{o}
\end{aligned}
$$

This angle is between the flow of the core of the vortex and its environment. If the ambient air is flowing down, or if a disturbance is propogated against the side of the core of the vortex, Ø will not be the visible angle measured in a stationary coordinate system.

In the experiments, vortex instabilities were generated by passing a flat plate downward near the fire whirl. Tyler (1931) has shown that a series of axial vortices are generated by flow over a flat plate, similar to von Karman vortices behind a cylinder. His measurements established that the dimensionless group ($f\ b \sin \xi / s$), where f = the frequency of the vortices, b is the chord of the plate, ξ is the angle of incidence and s is the velocity, has a value of 0.154 over the range of angles from 20^{o} to 90^{o} and a range of velocities from 0.2 to 4.5 meters per second. This equation can be solved for the characteristic size of one of the series of vortices, s/f:

$$s/f = (b \sin \xi / 0.154)/2$$

An additional theoretical result (Bessel, 1967) which will not be reproduced here, indicates that the influence of pertubations on vortices is much stronger upstream than downstream. As small disturbances move upstream (downwards in a fire whirl) they are compressed and preserved, while they are stretched and tend to decay rapidly as they move downstream (upwards). Thus, turbulent pertubations at the base of a fire whirl are quickly damped while disturbances some distance above the bottom of the whirl are preserved or amplified as it moves towards the ground.

An additional similarity requirement between atmospheric and laboratory generated vortices is that the flux Reynolds number, $Re_f = M/\nu r$, where ν is the kinematic viscosity, r is a length scale taken as the radius of the part of the vortex core in solid rotation, and M is the volume flux into the vortex given by:

$$M = \oint \vec{v} \cdot \vec{n}\, ds$$

(Greenspan, 1968). The rotational Reynolds number, Re_r, is $r\ v_t/\nu$ (Ying and Chang, 1970), where v_t is the tangential velocity at r.

The relation between these parameters given by

$$Re_t/Re_f = \tan \phi/\pi$$

is a requirement that the advance angles in laboratory and outdoors fire whirls and other vortices must be similar for the experiments to be similar.

FIRE WHIRLS OUTDOORS AND IN THE LABORATORY

Fire whirls occur frequently in wildland fire situations. They often move out of fire areas into hot burned-out areas and become dust devils. Conversely, dust devils occasionally move into fire areas and become fire whirls. Fire whirls with wind speeds of 50 meters per second (measured from streaks on photos) have been reported (Butler, 1969). Anemometers in fire areas have measured wind speeds of 30 meters per second (Palmer, 1970) and 25 meters per second (Wilson, 1971). The fire burned under relatively mild burning conditions. Much greater wind speeds have apparently occurred under proper conditions of terrain, wind and burning conditions (Pirske et al., 1965) with damage similar to that observed in tornadoes. A large outdoor fire whirl is one of the Project Flambeau experimental fires shown in Fig. 1.

The laboratory apparatus used in this study consisted of a vertical cylinder, 112 cm in diameter, slightly deformed into a spiral shape (Fig. 2). The fuel was burned in the center of the generator in a circular pool 12.7 cm in diameter which was flush with the floor of the generator. Experiments with pools of hot water gave visual results which were similar to the fire whirl experiments, but their photographs did not successfully define the fire whirls. The hood above the cylinder could be raised and lowered. Ventilation was provided by a blower in the exhaust duct, 6 meters above the apparatus. Photos were taken through the plexiglass window and over the top of the cylinder.

Pertubations were applied to the fire whirl by waving a flat plate near the top of the cylinder, tangentially to the whirl and directed downward. The flat plates were 5 cm x 15 cm, 10 cm x 20 cm, and 22 cm x 28 cm. The most effective size was 10 cm x 20 cm.

The pertubations had to be small. The most effective impulses were generated when the flat plate was held at an angle of incidence of about 30^o and moved with a downward speed of approximately 30 cm per second. Stronger impulses at larger angles of attack moved the fire whirl laterally, but had little influence upon it.

RESULTS

Dimensions of the large fire whirl in Fig. 1, are indicated by the simulated house in the fire which was 15 meters across. The advance angle was approximately 27^o relative to the axis of the fire whirl. The average value for three fire whirls was 26.8^o.

The advance angle of the undisturbed fire whirl in the laboratory is indicated in Fig. 3. The value of 28.4^0 was close to the average value of 26.8^0 of the outdoor fire whirls but approximately one-half of the 62.5^0 value necessary for instability. A number of turbulence generating devices were placed at the bottom of the fire whirl chamber. They included 8 cm x 8 cm x 16 cm blocks, screens of 0.64 cm mesh and rows of 2.4 cm rods, 20 cm high separated by 5 cm intervals. Variations in the exhaust blower strength increased the fuel evaporation rate and lengthened the fire whirl. None of these actions had any appreciable effect on the advance angle of the fire whirl.

The small pertubations introduced by the movement of the flat plate, as shown in Fig. 4, introduced significant instability into the fire whirls. An extreme example (Fig. 5) was a pertubation which developed into a vortex ring which propagated down the fire whirl almost to the bottom of the apparatus and nearly put out the flames. The pertubation apparently caused the fire whirl to bend over and around itself and form the vortex ring. This sort of pertubation could be formed about 10% of the time, while some pertubation was introduced about 80% of the time.

DISCUSSION AND CONCLUSIONS

The conditions of similarity between the laboratory fire whirls and the outdoor fire whirls were apparently satisfied in these experiments. By analogy, the results can be extended to other microscale vortices such as dust devils and possibly to mesoscale phenomena such as tornadoes.

The vortex instability resulted from a small pertubation in the flow field as a result of hydrodynamic instability. The instability was either: (1) a result of a local change in the advance angle so that it exceeded the critical value, or (2) the interaction between the horizontal vortex produced by the flat plate and the fire whirl vortex. The characteristic size of the most successful pertubations were computed to be:

$$v/f = (b \sin \xi)/ 0.158 \times 2$$

$$= 10 \times 0.5 / 0.158 \times 2$$

$$\simeq 15 \text{ cm}$$

This value is close to the size of the fire whirl, which indicates that hydrodynamic resonant instability may be the most important effect.

Two possible outdoor experiments are possible to examine the possibility of inducing pertubations on the atmospheric vortices. The first would use either a light airplane or helicopter wake vortex as the disturbing pertubation. This flight would have to be in a steep bank so that the vortices propagated toward the dust whirl. The second would use a large aircraft whose trailing vortices were on the same order of size as a tornado vortex. This flight would also have to be in a steep turn.

It is also possible that seeding the tornado vortex in a horizontal line could introduce a perturbation in the flow field which would disturb the storm sufficiently to destroy or damp the swirling flow.

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CAPTIONS

Fig. 1 A large outdoor fire whirl in an experimental mass fire. The simulated house is 15 meters across. The advance angle above the surface inflow is approximately 27^{o}. Ambient winds caused it to lean to the right.

Fig. 2 The laboratory fire whirl generator.

Fig. 3 An undisturbed laboratory fire whirl with an advance angle of 28^{o}.

Fig. 4 Geometrical relation between the firewhirl vortex and the flat plate generated vortex in the laboratory experiments.

Fig. 5 Vortex instability progressing upstream as a result of a weak pertubation.

FIGURE 1

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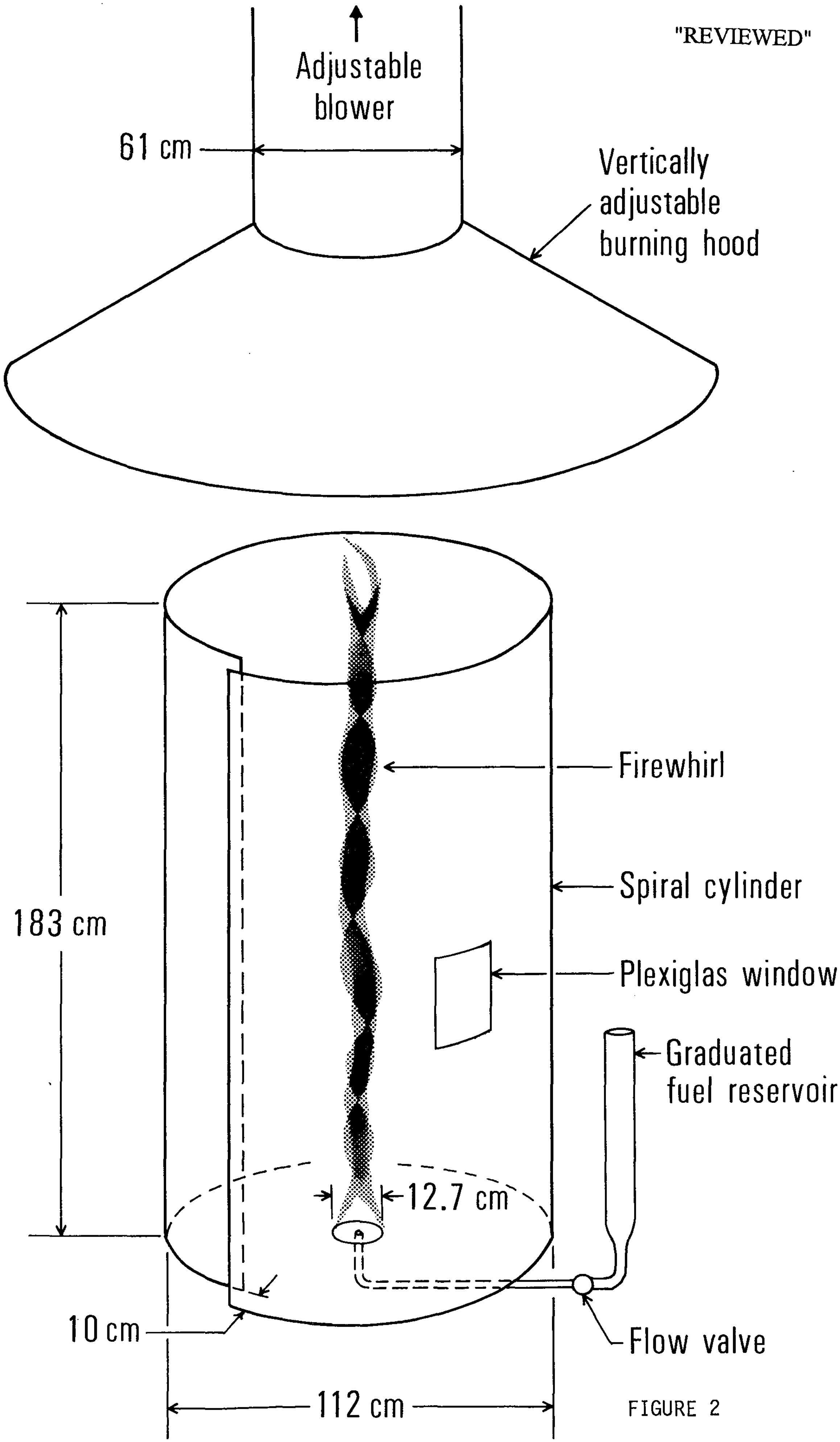

FIGURE 2

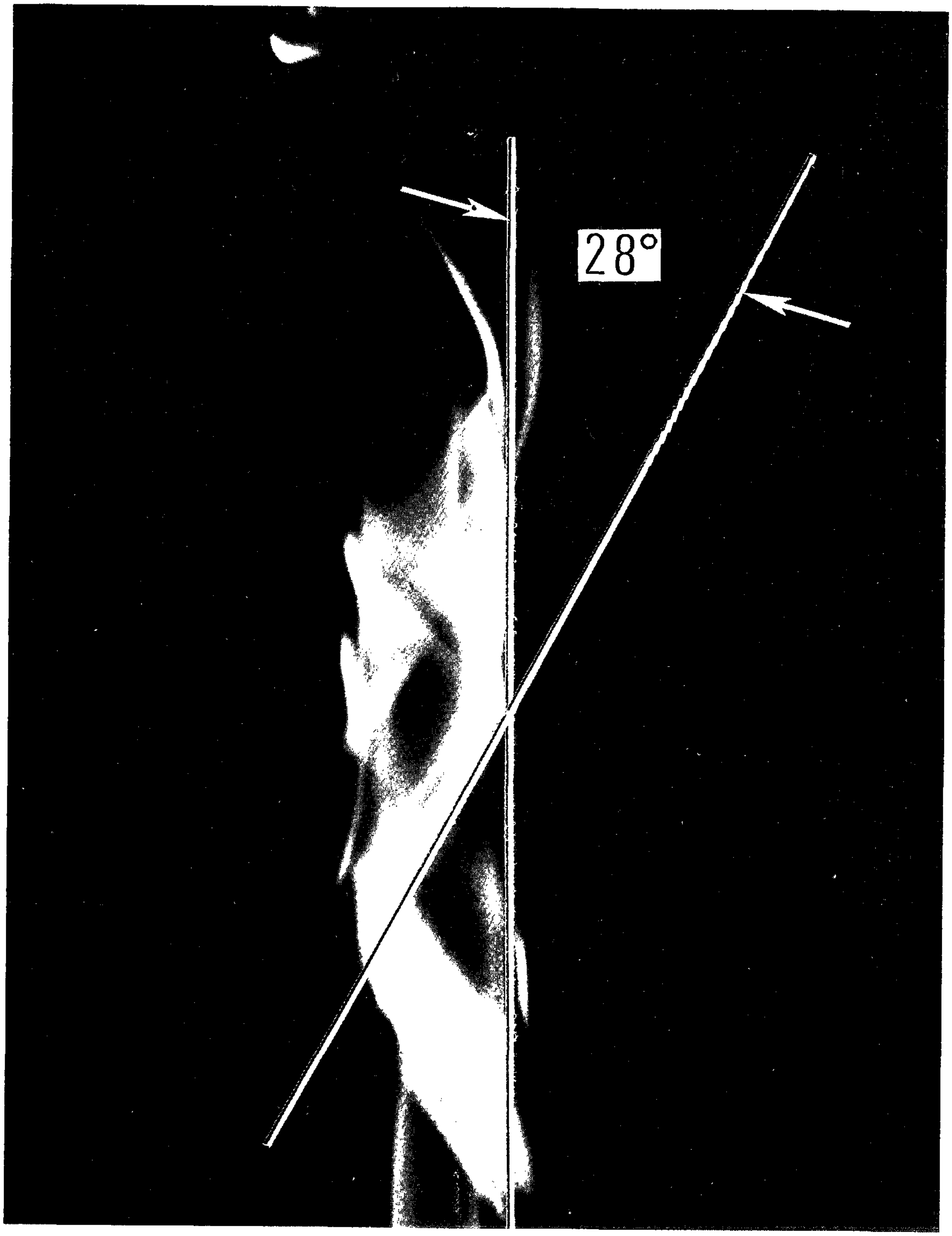

FIGURE 3

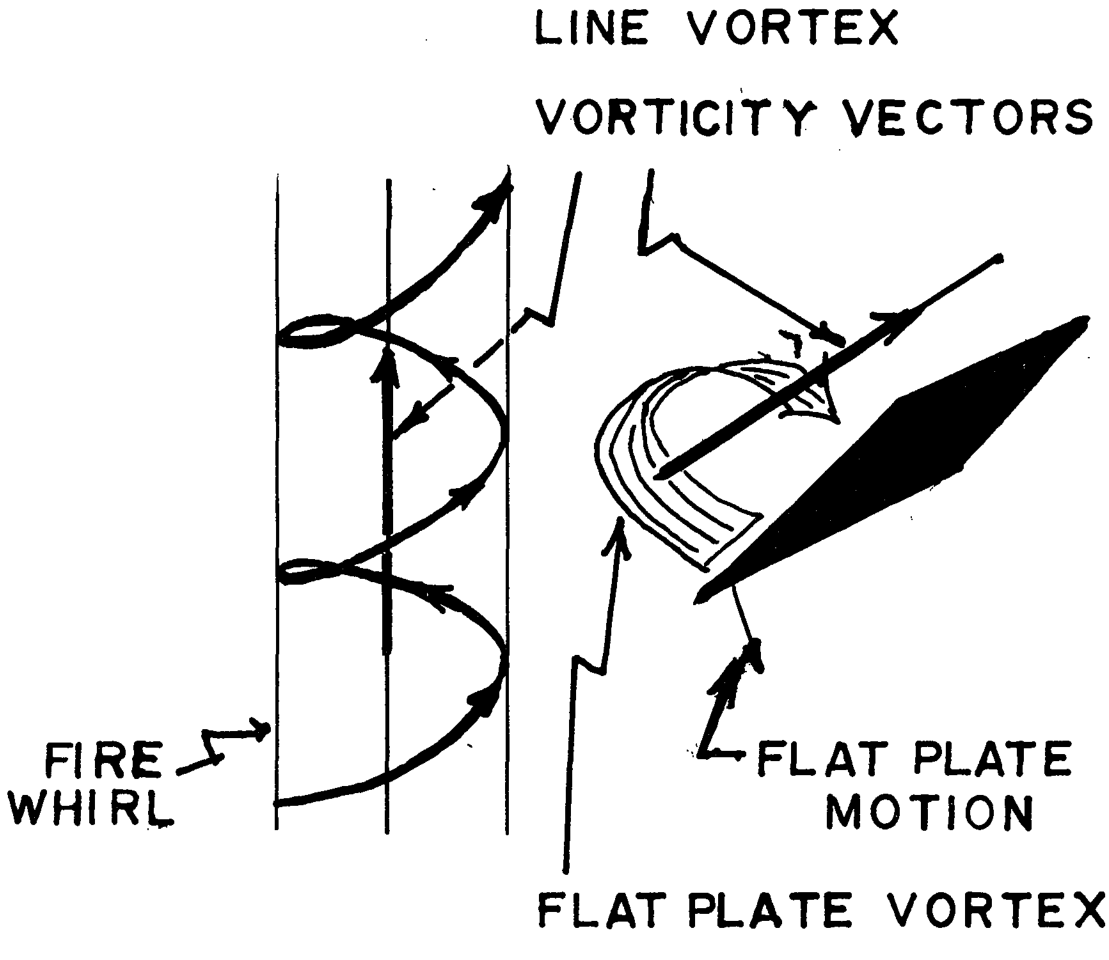
LINE VORTEX
VORTICITY VECTORS
FIRE WHIRL
FLAT PLATE MOTION
FLAT PLATE VORTEX

FIGURE 4

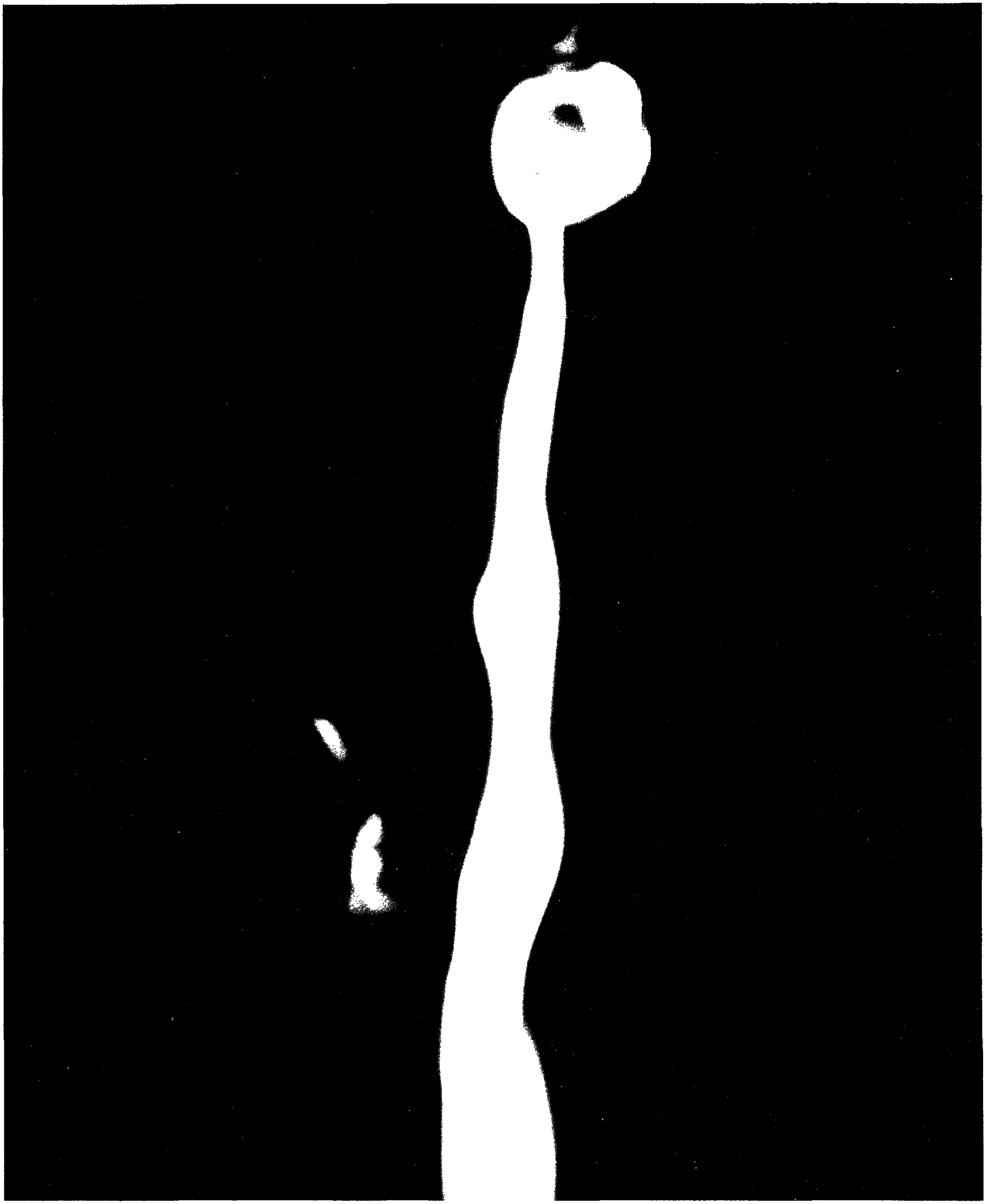

FIGURE 5